Binary Classification Using the Sugeno Integral

Sadegh Abbaszadeh and Eyke Huellermeier

Paderborn University, Germany

sadegh.abbaszadeh@uni-paderborn.de,eyke@upb.de

The idea of combining models and aggregation functions from the field of (multiple-criteria) decision making with data-driven approaches for model identification from the field of machine learning has attracted increasing attention in recent years. Examples of such combinations include methods for learning the majority rule model [2], the non-compensatory sorting model [3], or the Choquet integral [5]. In contrast to many other machine learning approaches, correspond- ing models are interpretable and meaningful from of decision making point of view. Besides, they often guarantee other properties that might be desirable, such as monotonicity.

The general structure of such models is as follows: Given a choice alternative described in terms of an attribute vector, each attribute is first evaluated by means of a local utility function, and thereby turned into a local utility degree. In a second step, the local utility degrees are aggregated into a global utility. Finally, a decision or an action is taken based on this utility.

In this paper, we propose a method for binary classification, in which the aggregation is realized by the Sugeno integral [4], and decisions (classifications) are produced via thresholding. Due to the specific nature of the Sugeno integral, this approach is especially suitable for learning from ordinal data, which is a topic that has not received much attention in machine learning so far. More concretely, our contributions are as follows:

- We introduce the model class of threshold classifiers based on the Sugeno integral and analyze theoretical properties of this class. In particular, we quantify its expressivity in terms of the VC-dimension, a measure that plays an important role in statistical learning theory.
- We formalize the problem of learning corresponding models as a problem of empirical risk minimization. This problem essentially comes down to learning the capacity underlying the Sugeno integral. To this end, we develop an algorithm based on linear programming.
- As part of the learning algorithm, we also propose suitable normalization techniques for turning attribute values into local utilities, as well as a method for tuning the threshold for the final (binary) decision.
- To control the flexibility of the classifier and mitigate the problem of overfitting the training data, we generalize our approach toward k-maxitive capacities [1], where k plays the role of a hyperparameter of the learner that is automatically tuned to the data at hand.
- Finally, we present experimental studies in which we compare our method with competing approaches on several benchmark data sets.

References:

[1] R. Mesiar and A. Kolesarova. k-maxitive aggregation functions. Fuzzy Sets and Systems, 346:127-137, 2018.

[2] Olivier Sobrie, Vincent Mousseau, and Marc Pirlot. Learning a majority rule model from large sets of assignment examples. In Proc. ADT, 3rd International Conference on Algorithmic Decision Theory, pages 336-350, Bruxelles, Belgium, 2013.

[3] Olivier Sobrie, Vincent Mousseau, and Marc Pirlot. Learning the parameters of a non compensatory sorting model. In Proc. ADT, 4th International Conference on Algorithmic Decision Theory, pages 153-170, Lexington, KY, USA, 2015.

[4] M. Sugeno. Theory of Fuzzy Integrals and its Application. PhD thesis, Tokyo Institute of Technology, 1974.

[5] A. Fallah Tehrani, W. Cheng, K. Dembczynski, and E. Hullermeier. Learning monotone nonlinear models using the Choquet integral. Machine Learning, 89:183-211, 2012.

RU-Implications Satisfying U-Modus Tollens

Isabel Aguilo Pons, Juan Vicente Riera and Joan Torrens

University of the Balearic Islands, Spain

{isabel.aguilo,jvicente.riera,jts224}@uib.es

The generalized Modus Tollens inference rule is normally used in fuzzy logic and approximate reasoning to make backward inferences similarly as the generalized Modus Ponens is used to make forward inferences. Although the Modus Ponens has been extensively studied in the literature, only some few papers have appeared dealing with the Modus Tollens.

In the fuzzy logic framework, both inference rules lead to different inequalities involving a conjunction, an implication, and a negation (in the case of Modus Tollens). Usually a t-norm is considered as the conjunction, but recently, the possibility of changing the t-norm by a conjunctive uninorm in the Modus Ponens has been studied, leading to the so-called U-Modus Ponens. Then, it is proved in [1] that the most adequate implications in this case are those derived from uninorms, specially residuated implications or RU-implications.

In this paper we want to deal with a similar generalization of the Modus Tollens, changing again the t-norm by a conjunctive uninorm, leading to the U-Modus Tollens. In this direction, given a uninorm U, a negation N, and an implication I, some general results are proved giving necessary conditions on U,N, I in order to satisfy the U-Modus Tollens. Then, some special cases are considered depending on how are the involved operators. Specifically:

- All solutions of the U-Modus Tollens are characterized in the case when U is a uninorm in Umin, I satisfies the neutrality principle, and N is strictly decreasing.
- The case when I is an RU-implication is studied by considering three different possibilities separately. Namely, when I is a residuated implication derived from a uninorm in Umin, when I is a residuated implication derived from a conjunctive representable uninorm, and finally when I is a residuated implication derived from a conjunctive idempotent uninorm. It is proved that there are a lot of solutions of the U-Modus Tollens in all these cases.

As a future work, many other possibilities can be explored, like residuated implication derived from other kinds of conjunctive uninorms, or material implications derived from disjunctive uninorms, or even h-generated implications.

Acknowledgement:

This work has been partially supported by the Spanish Grant TIN2016-75404-P AEI/FEDER, UE.

References:

[1] M. Mas, D. Ruiz-Aguilera, J. Torrens, Uninorm based residual implications satisfying the Modus Ponens property with respect to a uninorm. Fuzzy Sets and Systems, 359, 22-41 (2019).