FUZZY HIT-OR-MISS TRANSFORM USING UNINORMS

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GENERAL IDEA

Design

Fuzzy Hit-or-Miss operator using implications derived from uninorms.

Study

Its theoretical properties.

Analyze

Experimental results.

Preliminaries

Logical Operators

Formal Description of Images

Fuzzy Mathematical Morphology

Fuzzy Hit-or-Miss transform

LOGICAL OPERATORS

Definition

A non-decreasing binary operator $C : [0,1]^2 \to [0,1]$ is called a *fuzzy conjunction* if it satisfies $C(0,1) = C(1,0) = 0$ and $C(1,1) = 1$.

Definition

A fuzzy conjunction T on $[0,1]$ is called a *t-norm* when it is commutative, associative and it satisfies $T(1,x) = x$ for all $x \in [0,1]$.

LOGICAL OPERATORS

Definition

A binary operator $I : [0,1]^2 \to [0,1]$ is a *fuzzy implication function* if it is non-increasing in the first variable, non-decreasing in the second one and it satisfies $I(0,0) = I(1,1) = 1$ and $I(1,0) = 0$.

Definition

A non-increasing function $N : [0,1] \to [0,1]$ is called a *strong fuzzy negation* if it is an involution, i.e., if $N(N(x)) = x$ for all $x \in [0,1]$.

Definition

A non-decreasing binary operator $U : [0,1]^2 \to [0,1]$ is a *uninorm* if it is associative, commutative and there exists $e \in [0,1]$ such that $U(x,e) = U(e,x) = x$ for all $x \in [0,1]$.

- U is *conjunctive* if $U(0,1) = 0$.
- U is *idempotent* if $U(x,x) = x$ for all $x \in [0,1]$.
- U is *representable* if is conjunctive and there exists a continuous, strictly increasing function $h : [0,1] \to [-\infty,+\infty]$, with $h(0) = -\infty$, $h(e) = 0$ and $h(1) = +\infty$ such that $U_h(x,y) = h^{-1}(h(x) + h(y))$ for all $(x,y) \in [0,1]^2 \setminus \{(0,1),(1,0)\}$ and $U(0,1) = U(1,0) = 0$.

Definition

Given a fuzzy conjunction C such that $C(1, x) > 0$ for all $x > 0$, the binary operator

$$I_C(x, y) = \sup\{z \in [0, 1] \mid C(x, z) \leq y\}.$$

Implications from uninorms

All representable uninorms admit a residuated fuzzy implication function.

Definition

A *multivariate image*, A, is a map $A : \mathbb{Z}^n \to \mathscr{C}_1 \times \ldots \times \mathscr{C}_m$, where $\mathscr{C}_i \subset \mathbb{R}$ are its channels.

A *structuring element*, B, is a one-channel image.

We consider that $\mathscr{C}_1 = [0, 1]$.

The *spatial translation* by a vector $v \in \mathbb{Z}^n$, T_v, is a map such that

$$\forall d \subset \mathbb{Z}^n, \forall a \in \mathbb{Z}^n, \quad a \in T_v(d) \iff a - v \in d.$$

Definition

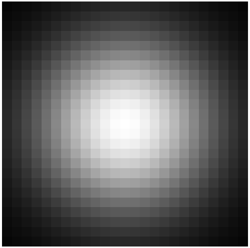
$$
\begin{aligned}
\text{Dilation:} \quad A \oplus B &= \textstyle\bigcup_{b \in B} A_b, \\
\text{Erosion:} \quad A \ominus B &= \{x \ : \ B_x \subseteq A\}.
\end{aligned}
$$

Definition

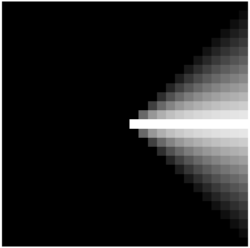
$$\begin{aligned}
\text{Dilation:} \quad \mathcal{D}_C(A, B)(y) &= \sup_{x \in d_A \cap T_y(d_B)} C\big(B(x-y), A(x)\big), \\
\text{Erosion:} \quad \mathcal{E}_I(A, B)(y) &= \inf_{x \in d_A \cap T_y(d_B)} I\big(B(x-y), A(x)\big).
\end{aligned}$$

FUZZY MATHEMATICAL MORPHOLOGY

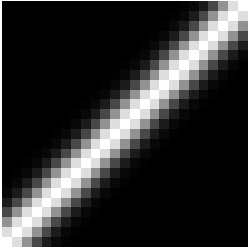
GREY-SCALE STRUCTURING ELEMENTS



Being close to one object.



Being to the right of one object.



Being in a diagonal layout.

General idea

Two structuring elements, B_1, B_2.

Hit: B_1 included in the image.

Miss: B_2 excluded from the image.

Definition

Let A be a binary image and $B = (B_1, B_2)$ a pair of binary structuring elements. Then, the *binary hit-or-miss transform* of A by B is

$$A \circledast B = \{x \ : \ (B_1)_x \subseteq A, \ (B_2)_x \subseteq A^c\} = (A \ominus B_1) \cap (A^c \ominus B_2).$$

Definition

Let N be a strong fuzzy negation, I a fuzzy implication function and C a fuzzy conjunction. The *fuzzy Hit-or-Miss transform* ($\mathcal{FHM}$) of the grey-level fuzzy image A with respect to the grey-scale structuring element $B = (B_1, B_2)$ is defined by

$$\mathcal{FHM}_{C,I,N}(A,B)(y) = C\big(\mathcal{E}_I(A,B_1)(y), \mathcal{E}_I(\mathcal{N}(A),B_2)(y)\big),$$

where $\mathcal{N}(A)(x) = N(A(x))$ for all $x \in d_A$ and $C(A,B)(x) = C(A(x),B(x))$, for all $x \in d_A \cap d_B$.

How?

Let U be a conjunctibe uninorm. Then, it admits the residuated fuzzy implication function I_U.

$$\mathcal{FHM}_{C,I_U,N}(A,B)(y) = C\big(\mathcal{E}_{I_U}(A,B_1)(y), \mathcal{E}_{I_U}(\mathcal{N}(A),B_2)(y)\big).$$

$$\mathcal{FHM}_{U,I_U,N}(A,B)(y) = U\big(\mathcal{E}_{I_U}(A,B_1)(y), \mathcal{E}_{I_U}(\mathcal{N}(A),B_2)(y)\big).$$

THEORETICAL PROPERTIES

High output if shape present

Low Output if shape *not* present

Output for uniform regions

HIGH OUTPUT IF SHAPE PRESENT

Theorem

Let N be a strong negation, $B = (B_1, B_2)$ be a grey-scale structuring element where $B_2 = N(B_1)$ and B_1 is a part of A at the point y. Let U be a conjunctive uninorm with neutral element $e \in [0, 1]$. If one of the following two cases hold:

- *U is a representable uninorm, or*
- *U is a left-continuous idempotent uninorm $U \equiv \langle N, e \rangle_{ide}$ and there is a point t such that $B(t) = e$;*

then $FMHMT_{U,I_U,N}(A, B)(y) = e$.

LOW OUTPUT IF SHAPE *NOT* PRESENT

Theorem

Let T be a t-norm, N a strong negation and U be a conjunctive representable uninorm. If A is a grey level image and B_1 is a structuring element that is not a part of A. Then for all points $y \in d_A$,

$$FMHMT_{T,I_U,N}(A, B)(y) \leq e.$$

Theorem

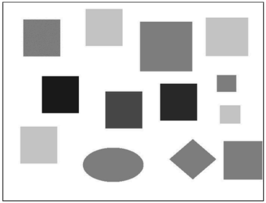
Let T be a t-norm, $U = \langle \ln \frac{x}{1-x}, e\rangle_{rep}$, I_U its R-implication, $N(x) = 1 - x$, A a grey-level image, $B = (B_1, N(B_1))$ a pair of structuring elements and $y \in d_A$. Let's suppose that $B_1(x) = m$ for all $x \in d_{B_1}$ and that $A(x) = k$ for all $x \in d_{T_y(B_1)}$, with $m < k$. Then,

$$FMHMT_{T,I_U,N}(A,B)(y) = \begin{cases} \min\left(\frac{k-km}{k+m-2km}, \frac{m-km}{k+m-2km}\right), & \text{if } T = T_{\mathbf{M}}, \\ \frac{(1-k)(1-m)km}{(k+m-2km)^2}, & \text{if } T = T_{\mathbf{P}}, \\ 0, & \text{if } T = T_{\mathbf{LK}}, \\ 0, & \text{if } T = T_{\mathbf{nM}}. \end{cases}$$

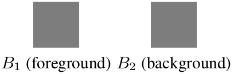
EXPERIMENTAL RESULTS

EXPERIMENTATION (I)

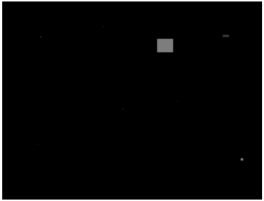
$b_1 = b_2$ FOR GREY SHAPES



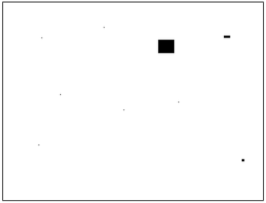
(a) Original image



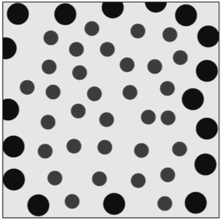
(b) Structuring element



(c) Fuzzy Hit-or-Miss transform

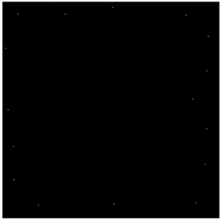


(d) Thresholded Fuzzy Hit-or-Miss transform

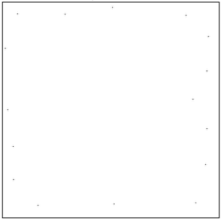


(a) Original image

(b) Structuring element



(c) Fuzzy Hit-or-Miss transform



(d) Thresholded Fuzzy Hit-or-Miss transform

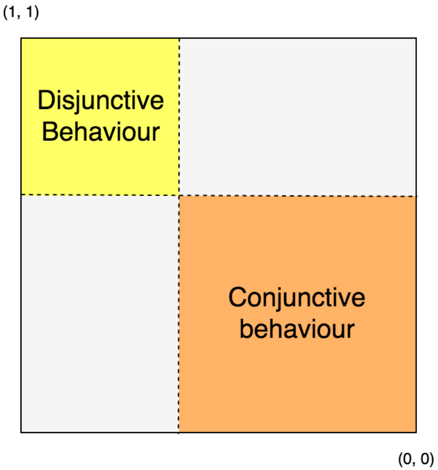
Conclusions

Analysis and Remarks

Future Work

ANALYSIS AND REMARKS

- ✓ Gray-scale templates B_1, B_2.
- ✓ Fuzzy approach.
  - Interpretable degrees of *fitting*.
  - Well-defined complementary of images.
- ✓ Expressive power vs computationally requirements.



(1, 1)
Disjunctive Behaviour
Conjunctive behaviour
(0, 0)

FUTURE WORK

- Best uninorms based on experimental results.
- Grey-scale images $\rightarrow$ Color images.

ACKNOWLEDGMENTS

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Thank you